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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2016

FIRST YEAR [BATCH 2016-19]

Date : 19/12/2016

MATHEMATICS FOR ECONOMICS [General]

Time : 11 am – 2 pm

Paper : I

Full Marks : 75

[Use a separate Answer Book for each Group]

<u>Group – A</u>

Answer ***any seven*** questions from **Question No. 1 to 11**: [7×5]

1. $A=\{x\in\mathbb{R}\mid x\in[2,5]\}$ and $B=\{x\in\mathbb{R}\mid x\in(2,5)\}$ then $\left(A^c-B^c\right)^c=?$ [5]

2. a) Define injective and surjective mapping from a set X to another set Y. [3]
   b) $f(x)=x^2-1, x\in\mathbb{Z}$, where $\mathbb{Z}$ is the set of integers. Is $f(x)$ injective? [2]

3. a) State Archimedean property and Density property of real numbers. [2]
   b) Assuming that between any two real numbers $\exists$ a rational number, prove that between any two real numbers there also exist an irrational number. [3]

4. a) Prove that Union of two open sets is open. [3]
   b) Give an example to show that intersection of an infinite collection of open sets may not be an open set. [2]

5. Find the set of limit points of the following subsets of $\mathbb{R}$:
   a) The set of natural numbers. [1]
   b) $\left\{m+\dfrac{1}{n}\mid m\in\mathbb{N}, n\in\mathbb{N}\right\}$, where $\mathbb{N}$ is set of natural numbers. [2]
   c) $\left\{x\in\mathbb{R}\mid x^2<1\right\}$. [2]

6. Test the convergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n\cdot 2^n}$. [5]

7. a) Prove that if $\displaystyle\sum_{n=1}^{\infty}a_n$ is convergent then $\displaystyle\lim_{n\to\infty}a_n=0$. [3]
   b) Is the converse of the above statement true i.e. if $\displaystyle\lim_{n\to\infty}a_n=0$ then whether $\displaystyle\sum_{n=1}^{\infty}a_n$ is convergent. [5]

8. Test whether the series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{(n+1)^n}$ is convergent or not. [5]

9. a) Prove that every Cauchy sequence is a bounded sequence in $\mathbb{R}$. [3]
   b) Is the converse of the above statement is true? Justify. [2]

10. Show that the sequence $\left\{\frac{3n+1}{n+2}\right\}$ is monotone increasing. Is the sequence convergent? Justify your answer. If convergent, then find it's limit. [5]

11. Define Cauchy sequence. Prove that every convergent sequence is Cauchy sequence. [5]

<u>Group – B</u>

Answer ***<u>any four</u>*** questions from **<u>Question No. 12 to 17</u>** : [4×5]

12. a) If n be a positive integer, prove that $(1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}} \cos\frac{n\pi}{4}$. [3]

    b) Let the operation '*' be defined on $\mathbb{Z}$ (set of all integers) by $a*b = a+2b,\ \forall\ a,b \in \mathbb{Z}$. Does there exist an identity element? If exist find the identity element. [2]

13. a) Show that the ring of matrices $S = \left\{\begin{pmatrix} a & b \\ 2b & a \end{pmatrix},\ a,b \in \mathbb{R}\right\}$ does not form a field. ($\mathbb{R}$ is the set of real numbers) [3]

    b) Prove that the matrix $\frac{1}{3}\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{pmatrix}$ is orthogonal matrix. [2]

14. a) If A and B are two orthogonal matrices of the same order then check whether

    (i) (A+B) is orthogonal or not?

    (ii) (A^T B) is orthogonal or not? [1½+1½]

    b) Find the rank of the matrix $\begin{pmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{pmatrix}$. [2]

15. a) Define a non-singular matrix. [1]

    b) Is the sum of two non-singular matrix is non-singular? Justify. [2]

    c) If $x+y+z=0$, prove that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = 0$. [2]

16. Reduce the matrix $A = \begin{pmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{pmatrix}$ to the normal form and hence determine it's rank. [5]

17. a) Find the value of $\frac{1}{E+2}(5x+3)$, where E is the shift operator taking h = 1. [3]

b) Prove that for a polynomial u_x of degree n in x, Δu_x will be a polynomial in x of degree n–1. [2]

Answer ***any two*** questions from **Question No. 18 to 20** : [2×10]

18. a) Solve the difference equation $u_{x+2}-8u_{x+1}+25u_x=2x^2+x+1$. [6]

b) Solve the equation $x^6-1=0$. [4]

19. a) Consider the set Q' of all rational numbers except '–1' and a binary operation '*' on Q' defined by $a*b=a+b+ab\ \forall\ a,b\in Q'$. Prove that $(Q',*)$ is a group. [6]

b) Check whether the set $\left\{a+b\sqrt{5}\,|\,a,b\in\mathbb{Z}\right\}$ forms a field? ($\mathbb{Z}$ is the set of all integers) [4]

20. a) The first term of a sequence is 1, the second is 2, and every other term is the sum of the two preceding terms. Find the nth term. [5]

b) Prove that a real square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew symmetric matrix. [5]

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